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AUTHOR	Braden, Barbara; And Others
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ABSTRACT

This handbook, the result of a study begun in 1968, is designed to assist the forecaster in developing an increased sensitivity to potential changes in conditions affecting enrollments. Study results indicate that school enrollment forecasting is enhanced primarily by (1) knowing the community involved thoroughly, (2) applying a standard forecasting method regularly and often, and (3) assessing the probable error of those forecasts. Material in three sections of the document assists the forecaster by describing information needed to understand thoroughly community influences on school enrollments. In another section, a standard forecasting method known as cohort or percentage survival is described and step-by-step procedures for using that method are presented in a subsequent section. Assistance for use in determining the probable error of this method is provided in the last section, which presents detailed instructions for determining confidence intervals and forms to aid in the computations. (Author/JH)

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NESDEC

NEW ENGLAND SCHOOL DEVELOPMENT COUNCIL

ENROLLMENT FORECASTING HANDBOOK

introducing

Confidence Limit Computations for A Cohort-Survival Technique

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New England School Development Council
55 Chapel Street
Newton, Massachusetts 02160

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PREFACE

Enrollment forecasting is a basic ingredient in school system planning. Many projective methods have been used by many planners with varying results under various conditions. Measuring the inaccuracies associated with those methods and conditions, determining some of the reasons therefor and applying this information to improving the art of forecasting was the initial goal of a study begun in 1968 by staff members of the New England School Development Council with the assistance of Arthur D. Little Inc. The study was sponsored by Educational Facilities Laboratories, Inc.

The study led to the general conclusion that large differences between forecasts and subsequent actual enrollments are more often related to changing local trends and conditions than to specific methods applied, more often a function of the "clairvoyance" of the planner than of his technical prowess. To improve the art of forecasting then becomes a matter of increasing the sensitivity of the planner to potential changes in conditions affecting enrollments. This handbook is designed to assist the forecaster in developing that increased sensitivity.

The study led also to an awareness that communities varied greatly in the stability of their growth, some apparently growing steadily and predictably, others varying erratically, even though their general growth pattern might be similar to that of their more stable neighbors. It was apparent also that predictions for any one year for the stable community were apt to be found more nearly correct than they were for the erratic one. And no projection method appeared to be immune to this problem.

Unable to find ways of eliminating this phenomena, it then appeared wise to measure it and to use the knowledge to add a dimension of reality to understanding the nature of projections that much more. The search for a suitable measure led the statistically oriented members of the team to the application of confidence interval techniques to support this goal. The handbook presents the results of their search.

Confidence interval technology was developed by Stefan Peters of ADL, but many other people played key roles in producing this handbook. ADL's Donald Meals and NESDEC's John Sullivan examined and weighed the merits of many projection methods. John Sullivan field tested the initial procedures for computing confidence intervals. Revised on the basis of this experience, the materials were tested again and again until all were satisfied that the techniques and the directions were as sound as the theory on which they were based.

NESDEC's Simpson Fellow (1970-71), Barbara Braden, prepared the initial draft of the handbook and assisted, along with Richard Willard and other members of the staff, with final review and editing.

NESDEC wishes to thank Educational Facilities Laboratories, Inc. for the study grant which led to publication of this work. It also wishes to acknowledge the special assistance of Harold Gores and Alan Green of EFL. Their encouragement, criticism, and advice are greatly appreciated.

Robert S. Ireland
Executive Secretary
New England School Development Council

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I. INTRODUCTION

The art of forecasting is enhanced primarily by (1) knowing the community involved thoroughly—its history, its trends, its eccentricities, its plans; (2) applying a standard forecasting method regularly and often; and (3) assessing the probable error of those forecasts.

The need for the forecaster to know the community in which he is plying his trade is as obvious as the fact that he must be thoroughly familiar with factors influencing school enrollments. He must know the community's history, sample its atmosphere, taste its ambitions, measure its economic potential, savor its human resources, learn of its plans, measure its trends. Some of the information required to accomplish this goal may be easily found. Some is illusory. But there are ways of increasing one's understanding of even the illusory factors. Sections 4, 5 and 7 are designed to assist in this effort.

The standard forecasting method chosen for presentation in this handbook is the cohort-survival or percentage survival method. A method familiar to many schoolmen, it not only has a record for reliability in relatively stable districts, but the necessary calculations are simple and straightforward, the data requirements reasonable and usually easily fulfilled. Unless the district has undergone unusual or complicated growth patterns in recent years, reasonably good results can be expected. The only assumption of the cohort-survival method is that the net effect of factors influencing enrollments—migration, school policies, mortality, nonpublic school attendance—remain almost constant. And even if these conditions vary somewhat, certain modifications can be made to accommodate them. Space does not permit identification of all exceptions or unusual situations but, hopefully, an understanding of the basic procedures presented here will make judicious modifications possible. A brief description of the cohort-survival method follows in Section 2. Applying such a method regularly provides a means of becoming sensitive to changes within a community soon after they occur and adds another way that the planner can know his community. Over a period of time he will also become familiar with the biases and eccentricities of the technique employed. Step by step procedures for applying the cohort-survival method in projecting enrollments begin in Section 6. Forms are provided to simplify the procedure.

Planning school facilities and programs must always be accomplished in the face of uncertainties: uncertainties of future enrollments, future instructional techniques, economic and fiscal conditions, and public support. Uncertainty about future enrollments can be reduced through precise and regular collection of data, regular analysis of trends, identification of the major factors influencing change, judicious selection of procedures for making projections, and through careful application of those procedures. But uncertainty cannot be eliminated, and measurement of the degree of uncertainty is a goal that responsible decision-makers find desirable. This can be done, at least in part. Statisticians like to deal with two kinds of uncertainties or variations in data. One they call random variations; the other, systematic. No way has yet been determined to measure the probable range of errors in forecasts due to errors in initial premises or in systematic changes in conditions not already present in past experience. These are the kinds of errors that only a thorough knowledge or the community and projection methods employed can guard against. However, random error, the variability of forecasts that might be deduced from a knowledge of past history, can be estimated. It is that part of uncertainty that is treated in this handbook with confidence intervals, a valuable addition to the tool kit of the educational planner. This treatment involves determining a range of enrollments within which the true value of a forecast will probably lie, thus providing the means for a more realistic interpretation of estimates. Section 8 presents detailed instructions for determining confidence intervals and forms to aid in the computations.

Although the computations for confidence limits may be rather tedious, they can be done "by hand" with the aid of a desk calculator. Although they may also be done by computer, it is suggested that the forecaster will become acquainted with the technique and its meaning more quickly by performing the computations initially by hand. Later, use of the computer, both for the projections and for calculating confidence intervals will greatly reduce the labor involved.*

*NESDEC provides a service to turn raw data into projections with confidence intervals using the procedures outlined in this handbook; the computer program was prepared by Richard Willard.

II. THE COHORT-SURVIVAL METHOD

The basic assumption of the cohort-survival method for predicting school enrollments is that what has happened in the past will continue to happen in the future; that is, given the number of births, the net effect of all other influences on enrollment will remain proportionately the same.

The basic technique requires calculating the ratio of the number of children in one grade in one year compared to the number of children who "survive"* the year and enroll in the next grade the following year. Fluctuations in such data from year to year create a pattern from which an average survival rate can be calculated to project an enrollment. Thus, if over a period of ten years, an average of 96 percent of the enrollment in Grade 3 goes on to Grade 4 and if 300 children are now enrolled in Grade 3, then next year's Grade 4 enrollment may be estimated at 96 percent of 300, or 288 students. Eleven average rates of survival are calculated for a system with twelve grades. These rates can then be applied to the present enrollment and used to project enrollments each succeeding year. Thus, if the average survival rate from Grade 4 (with its 288 students) to Grade 5 is 1.10, then for the second projected year the estimate for Grade 5 is 1.10 of 288, or 317 students.

Of course, forecasts for successive years must take as their starting point an estimate of the number of children entering kindergarten or first grade. These estimates may be made by methods similar to those mentioned. An average birth survival rate may be obtained by comparing known enrollments in kindergarten (or first grade) with birth data five (or six) years earlier. This rate may then be used to project enrollments for the initial school years from births. Thus, if an average birth survival rate—births to Grade 1—was found to be 1.17 or 117 percent in recent years, reflecting a net influx of preschool-age children, the planner could reasonably project future first grade enrollments from the number of recent births.

Since enrollment forecasts are a function of two variables, the number of births and the survival rates, reliance on numbers of births within a school district limits forecasts to relatively short-range projections for the lower grades. Usually only projections for the next four or five years are possible. To extend projections beyond this point, the future number of births must be estimated, compounding possible error by projecting from projections. (For those who

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*The term "survive" is enclosed in quotation marks to call attention to the specific use of the word in this situation. Children enrolled in the next grade, next year, are rarely an exact cohort of their previous grade and year. School promotion, migration, death may play a part. All these factors are included in the term "survival" as used in this context.

revise their forecasts each year or who need only short-term projections, extension of the forecasts by this means need not be a matter of concern.) There may also be limitations in data collection. A history of resident births may not be available or census boundaries may not coincide with school district boundaries. On the whole, however, and despite these limitations, the cohort-survival method remains one of the best possible means of forecasting enrollments.

III. CONFIDENCE INTERVALS

Knowing the range of probable values, or the *confidence interval*, within which enrollments may be expected to fall because of random variations, enables the planner to make estimates with graphic realism. The confidence interval is characteristically used to interpret test scores. A sophisticated psychometrist, for example, is reluctant to assign a single IQ score to a child since the random error associated with the test introduces reasonable doubts about its precision. He is more likely to say the chances are even that the true value lies within a certain range of scores. Technically, he is indicating that, due to random error, there is a 50-50 chance that the true value would fall within the stated range, or that the 50 percent *confidence interval* is marked by these limits. Similarly, confidence intervals may be applied to estimates of school enrollments.

Once the planner has selected the level of probability he desires for his projections, he can compute confidence intervals at that level. The interval is expressed as a range between a low estimate and high estimate within which an enrollment may be expected to fall given the chosen level of probability. For a degree of certainty, such as probability of .80, the difference between the high estimate and the low estimate will be large, but on the other hand, there is an 80 percent chance that the true enrollment will lie somewhere between these two numbers. Given a 50 percent probability, the range will be smaller; however, it is equally likely that the true enrollment will fall outside that range.

As an example, suppose that a specific set of data resulted in a cohort-survival projection of students in Grade 7 five years hence of 800, and computations of an .80 confidence interval using the same data resulted in an upper and lower limit of 938 and 682. This information may be interpreted that, due to random error, there is an 80 percent chance that the enrollment of that grade in that year will fall between 682 and 938. Similarly, if a confidence interval of .50 is selected for the same data, a smaller interval of 736 to 870 would result and be interpreted that there is a 50 percent chance that the true enrollment would lie within this range.

The warning previously noted should be repeated here. The determination of confidence intervals assumes that variations in enrollments have been due and will continue to be due to random changes in many factors influencing those enrollments and to their interaction upon each other. It is possible that systematic errors can be introduced at any time because of drastically altered conditions and circumstances. If they can be detected in the historical data it is possible that judicious modifications of the standard form of projection (see Section 7) may be made such that the systematic errors can be avoided. In other cases, one can obtain reliable forecasts only through detailed analyses of all factors and by anticipating and assessing the effects of future changes.

IV. FACTORS INFLUENCING FUTURE ENROLLMENTS

We have noted that the cohort-survival method assumes that the aggregate of all factors influencing past enrollments (except births) will continue to exert a similar influence in the future. While particular factors may vary appreciably from time to time, their total effect will tend to average out over an extended period. However, an understanding of these factors and their normal variations is essential for an effective use of the method. A major change in local school policies, zoning regulations, or the rate of industrial expansion might render the model inadequate and question its unqualified application. The careful planner must know his community, analyze the various factors known to influence school enrollments, and be aware of factors that may come into play in the future. He must then weigh their total effect: if the model appears to be reasonable, he should proceed to use it; if not, he must make appropriate adjustments or employ other procedures.

Factors influencing future enrollments in a specified school district may be classified into three categories—births, deaths, and migration.

BIRTHS

Not only are the number of births of first importance in projections, but obviously, births for the past several years are a major factor in determining primary enrollments for the next several years.*

People born in the baby boom following World War II will add their progeny to school enrollments in the seventies. (The lower number of births in the sixties is attributable, at least in part, to the shortage of young adults born in the thirties and early forties.) However, the present trend to limit family size, the advent of oral contraceptives, and legalized abortion appear to have significantly lowered projected birth rates for the seventies despite the increase anticipated from a growing number of marriages. Future trends are subject to many influences and wide fluctuations, as we shall see later on.

*Birth data for projection purposes are the number of live births to residents regardless of where the children are born. They are frequently called resident or allocated births. Enrollment forecasts beyond those four or five years call for a projection of future births. An understanding of the factors influencing births is thus essential. Economic conditions, social mores, and the general tenor of the times all influence birth rates in the population at large. Variations in ethnic, religious, and age composition of the local population may further differentiate rates between school districts.

DEATHS

On the other hand, mortality in a normal child population, barring major disaster, is usually small and relatively constant. In the cohort-survival method, the combined effect of death and migratory factors are expressed from grade to grade as survival rates.

MIGRATION

Since migratory habits may vary considerably from district to district, their analysis deserves careful attention. Even in the stable community, patterns may change abruptly. Only a persistent effort by the planner to know his community can guard against surprises.

School migration is affected primarily by the movement of families in and out of the community. This general migration is influenced by several factors: amount and types of available housing, amount of land available for development, zoning regulations, real estate values, local tax rates, availability of financing, highway development, attractiveness of the community, industrial growth or decline and job opportunities.

Changes in the amount and type of housing may be quickly reflected in the general population with only a remote effect on school enrollments. For example, 75 to 85 percent of new urban dwelling units are occupied by young couples with an immediate effect on adult population but a delayed impact on school enrollments. Similarly, apartment buildings may house many preschoolers but produce few school-age children as parents with growing families seek larger quarters.

Migration is also affected by school-related factors such as school programs, student-faculty relations, and alternative schools.

Good schools attract new residents and new students; new school buildings frequently have the same effect. On the other hand, double sessions may discourage new residents. The inauguration or elimination of highly relevant programs could influence migration from one community to another or give rise to alternative school systems within the given community. The quality of a school's program may also affect a student's choice to continue beyond the required compulsory attendance age. Kindergartens or nursery schools, transitional classes, non-graded, multi-graded, or open classroom plans may all be considered as evidence of program quality with consequent effect on migration. Similarly, schools reputed to be respecters of persons—both students and faculty—tend to retain larger portions of students in school after the compulsory attendance period is passed. A change in these relationships will undoubtedly affect the dropout rate and, although such changes tend to take place very slowly, when coupled with economic fluctuations or changes in job opportunities the effects could be appreciable.

The proximity of specialized schools (vocational-technical schools, for example) and the opening or closing of nonpublic schools may decisively alter migration patterns. The recent closing of many parochial schools has swelled enrollment in public schools. While most have phased out slowly—one or two grades at a time—in others, service has been cut off at all levels at once and the impact has been abrupt. For many years parochial school enrollments tended to be rather stable, being limited by whatever the local parish priest approved as optimum capacity. Recently, however, many have been subject to pressures from both parents and teachers to limit class sizes to numbers more nearly like those accepted in the public schools.

The difficulties of predicting changes such as these and foreseeing other influences to come emphasize the need to keep pace with community trends. One way for a planner to know his community is to collect and analyze relevant data regularly. He would also be well advised to keep lines of communication open to key citizens in the community.

V. DATA COLLECTION

Consistency is vital in collecting and using data. If "membership on October 1" is used initially as the statistic for enrollment, it would not be admissible to change midway to "average daily membership." Similarly, allocated births must be defined consistently and consistent geographic boundaries must be employed throughout the data-collecting period. Enrollments enumerated by grade may not normally be mixed with those counted by age.

If a community existed in which there was no migration then the children entering Grade 1 in September of a given year would include all the resident births of the calendar year six years prior to that event. In other words, the entering class and the appropriate births would represent the same cohort. The essence of the cohort survival method is that data used to determine each survival rate should represent the same cohort.

In actual practice, this idealized situation never exists. Families do move in and out and sometimes, particularly in high mobility areas, only a small fraction of those who enter Grade 1 this year will be found among those in Grade 2 next year. But in most cases others of the same age bracket and with similar attributes will have taken their places. For all practical considerations pupils progressing under these conditions may be considered as representing the same cohort and dealt with accordingly.

However, as a general rule, a good planner proceeds to make projections only when he knows he is dealing with, or has corrected to achieve, consistent data for a reasonably consistent cohort of children within consistent geographical boundaries.

ENROLLMENT DATA

Statistics used to represent enrollments in the cohort-survival method include average daily attendance (ADA), average daily membership (ADM), the frequently used enrollment on a fixed day (the school day nearest October 1), among others. Whichever is employed, it is easy to see that the projection must be interpreted in the same terms. For example, if membership statistics collected on October 1 of each of the last ten years were employed as basic data then, in turn, future estimates must refer to membership on October 1 of subsequent years.

Enrollment data may be obtained directly from the office of the superintendent of schools or, in most states, from statistics compiled by state departments of education.

BIRTH DATA

Since communities with the largest maternity hospitals will of course record the most births, most cities and towns routinely exchange such vital statistics so that all births, wherever they occur, are allocated to the community in which the baby's family resides. These statistics, usually summarized by calendar years, are normally appropriate for use for projection purposes.

Where municipalities and school districts are not coterminous, a common problem exists. Birth data is more often recorded by towns and cities (and political subdivisions of large cities) than by school districts. It may also be summarized for a period of time different from that governing school admissions. Although many valuable projections have been made from data whose geographical and time similarities, although consistent, were minimal, it must be recognized that the greater the degree of dissimilarity, the larger the possibility for error.* One other caution is appropriate. The allocation of births to residents of a community takes time, and officials in different communities vary in their efficiency in transmitting and recording such information. The result is that resident births are frequently unreliable for the latest six- to eight-month period.

Resident birth data is usually recorded in town and city annual reports; most recent data may be obtained from town clerks or from departments of vital statistics. Some states maintain such records for all political subdivisions.

BIRTH PROJECTIONS

Planners employ many methods of projecting births, some simple, some very complex. Among the simplest, and probably least accurate, are extending present trends (by graphical or arithmetic methods) or using the average of recent years. A preferred method employs projected national rates. Although the experts are known to have been wrong on occasion, the forecasts of the U.S. Census Bureau are probably the best scientifically based projections available for the country, and represent a consensus of the experts regarding sociological trends of the times. State forecasts may also be available from official sources and may be expected to reflect such patterns as may be peculiar to the state.

The U.S. Bureau of the Census regularly publishes three sets of estimates representing low, medium, and high birth rates. A comparison of local rates for the past five years or so with the national rates should provide some indication of the recent level of local fertility and, in the absence of evidence to the contrary, of probable levels for the future. (Please note that reference in this paragraph are to rates rather than numbers of births. For reasonable comparison the computation of local rates must be consistent with that of the national ones.) Another preferred method of estimating future births is to apply *relative change factors* estimated and published by the Bureau of the Census for the United States to the latest known actual births for the birth area under consideration. Current and projected national birth rates may be found

*It must be pointed out that demographers are not all in agreement about this point, some preferring to employ birth data from a larger area than the district under consideration. Their argument is that relationships between births in the greater area may be more consistently related to early grade enrollments than are births in the specified district, especially where migration is extensive. Under such conditions, the rise and fall of the number of births in the state or country may be a better predictor of local enrollments than local births.

in *Current Population Reports*, Series P-20 and P-25, U.S. Bureau of the Census. (If the planner is not fully experienced in this field, it is suggested that the services of a good demographic consultant can be invaluable.)

PRESCHOOL CENSUS

When no reliable birth records are available or applicable, it may be necessary to substitute information from a preschool census. In some states the practice is a yearly routine. Frequently it is coupled with a count of all school-age children and the collection of such other information as may be useful in "knowing the community."

Census information is equivalent to allocated birth data since the number of children under one year of age should approximate the allocated births for the previous twelve months. In addition, such data, collected annually, can be analyzed for indication of family migration patterns. If it includes, in addition, a preference for public or private schooling, the length of time the family has resided in the district, housing data, and similar information, much additional useful knowledge about the community may be acquired.

If neither birth data nor annual census records are available, a single preschool census, thoroughly and accurately done, can be used. A count of five-year-olds can serve as the basis for next year's first grade enrollment; four-year-olds for the following year's, etc. One must rely on other historical data (i.e., enrollments in both public and private schools) to make such adjustments as may be appropriate in actual projections. Procedures for conducting a census--the determination of questions to be asked, the selection and training of enumerators, the double checking of results—must be carefully developed and followed. (If the census includes school as well as preschool enumerations, one can check accuracy by comparing census data for a given grade with actual enrollments in that grade in the same district.) Even so, the results of such an approach can not be expected to be as reliable as projections based on consistent history and good birth data.

SUPPLEMENTARY DATA

Most supplementary data are used to obtain a knowledge of the community and determine the feasibility of the cohort-survival method under the circumstances. Data on land use, types of housing, zoning, assessed valuation and tax levels, as well as trends in housing construction, industrial or commercial growth, and highway development are among the factors about which one should have information in making such a judgment. This information is normally available from official agencies of local governments, assessors' offices, zoning boards, planning boards, public works and managers' offices. Maps, records, plans, and previous studies made by or for such agencies may be very useful. Frequently, such agencies can supply advance warning of impending changes. Bank officials, real estate agents, and public utility companies are among others who may possess valuable information.

Municipal governments and public utility companies frequently must make estimates of future demands and markets. An exchange of information and, often, an exchange of forecasts with such agencies can be mutually beneficial.

Trends in local economic conditions, tax rates, industrial growth, and job opportunities may be obtained from banks, chambers of commerce, commercial and industrial associations. However, one must be aware that such agencies are responsible for promotion as well as assessment and distinguish between the two. Regional, state, and national organizations may provide general economic trend information that may be pertinent to local situations as well.

VI. CALCULATING ENROLLMENT PROJECTIONS

The following step-by-step procedures for projecting enrollments are designed to initiate those new to the business of forecasting by the cohort-survival method. They fall into three categories: collecting data on enrollments and births (steps 1-2); calculating Average Survival Rates (steps 3-6); and applying survival rates to yield enrollment estimates (steps 7-9). Worksheets have been included to expedite the process. They have been designed for grade or age projections but, for simplicity, specific procedures are outlined for projection by grade group only. To project by age, as for nongraded schools, substitute "age" for "grade" each time it appears in the directions.

The example used to illustrate the procedures is based on ten years of kindergarten through twelfth grade enrollment data and fifteen years of data on the number of births. The five-year differential represents the normal time span between birth and enrollment in kindergarten. If the example were to start with Grade 1, birth data would be needed for sixteen years and one less Average Survival Rate, Kindergarten to Grade 1, would be included in the computation. (Instability in survival rates from birth to kindergarten sometimes suggests computing kindergarten enrollments separately.*) The birth column and the initial grade column (kindergarten or Grade 1) should contain data relating to the same cohort on the same horizontal line.

STEP 1

Collect historical data on enrollments by grade and year for the school system involved and record on Worksheet 1.

- a. *Starting with the current year (bottom of worksheet), number the years serially backwards from that point.*
- b. *Record resident births in the birth column corresponding to the calendar year indicated.*
- c. *Enter the grades by which enrollments are to be recorded in the top row of the Worksheet under PAST ENROLLMENTS.*

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*With many school systems, even though they provide kindergarten programs, it may be wise to choose Grade 1 as the initial grade in a projection. The non-application of compulsory attendance to kindergartens, the availability of private nursery and kindergarten schools and many other factors may tend to make kindergarten enrollments less stable than those of Grade 1. And since future estimates will be based on those of the initial year, the more stable base is desired. If Grade 1 is chosen as the initial year, kindergarten estimates may then be computed, backwards, from the Grade 1 data or, separately, from births.

d. *Record enrollments in appropriate boxes by grade and by the school year starting in September of the year indicated.*

(In the example the number of years of enrollment data, N, is 10 and birth data is collected for N + 5, or 15 years. If kindergartens are not included in the data, then birth data is needed for N + 6 years. With experience, step one may be eliminated in favor of recording the data directly on Worksheet 2.)

Example *Step 1*

PAST ENROLLM

age or grade (indicate

year*	births	K	I	II	III	IV	V	VI
58	391							
59	401							
60	392							
61	397							
62	399							
63	394	451	433	452	429	412	401	416
64	358	461	457	441	442	430	399	408

70	275	432	471	496	508	522	508	543
71	252	466	423	481	494	507	515	517
current year 72	N.A.	443	454	433	475	493	500	517

STEP 2

Transfer the information from Worksheet 1 to Worksheet 2. (The first worksheet simplifies data collection; the second, procedures for calculating.)

a. *Starting with the current year enter past years serially backwards in Column A in the upper section of Worksheet 2, PAST ENROLLMENTS.*

b. *Starting with the current year enter future years serially in Column A in the lower section of Worksheet 2, FUTURE ENROLLMENTS.*

c. *Enter birth dat in Column B (upper and lower sections of Worksheet 2). For projections involving kindergartens, use resident births five years prior to the year indicated in Column A.*
For projections starting with Grade 1, use resident births six years prior to the year indicated in Column A.

d. *Enter enrollment data in Column D for the appropriate years and grades.*

Example *Step 2*

A	B	C	D	E	D	E	D	E	D	E	D	E	D
school year commencing	births 5-6 yrs earlier*	B S R	age K		1		2		3		4		or 5
1963	386		451		433		452		429		412		401
1970	358		432		471		496		508		522		508
1971	341		466		423		481		494		507		515
current year 72	307		443		454		433		475		493		500
AVERAGE SURVIVAL RATES													
current year 73													
1973	327												
1974	290												

STEP 3

<u>Calculate Birth Survival Rates</u> by dividing each number of recorded births into the enrollment in kindergarten (Grade 1) representing the same cohort. In terms of Worksheet 2:

> *Divide each number in Column B into the number in Column D in the same row. Record the results in the intervening space in Column C. (Two decimal places are usually sufficient. For N years of data, N Birth Survival Rates will be calculated.)*

Example *Step 3* *Birth Survival Rate*

	A	B	C	D	E
	school year commencing	births 5-6 yrs earlier*	B S R	K	
ENROLLMENTS					
	1963	386	1.22	451	
	1964	391	1.18	461	
	1965	401	1.13	452	
	1966	392	1.17	458	
	1967	397	1.19	474	

451 ÷ 386 = 1.22

452 ÷ 401 = 1.13

STEP 4

<u>Calculate Grade Survival Rates</u> by dividing each grade enrollment into the enrollment of the subsequent grade in the following year. In terms of Worksheet 2:

> *Following the diagonal, divide each number of Column D into the number in the next Column D to the right, next row below. Record the result between the columns in the intervening boxes in Column E. Repeat for subsequent columns. Two decimal places are usually sufficient. (For N years of data, N-1 survival rates will be calculated.)*

Example *Step 4* *Grade Survival Rates*

WO

A	B	C	D	E	D	E	D	E	D	E
school year commencing	births 5-6 yrs earlier*	B S R	age K		1		2		3	
1963	386	1.22	451	1.01	433	1.02	452	.98	429	
1964	391	1.18	461	1.0[illegible]	457	.99	441	1.03	442	
1965	401	1.13	452	1.05	472	1.01	452	1.01	455	
1966	392	1.17	458	1.01	474	1.02	499	1.03	458	
1967	397	1.19	474	1.08	490		483		516	

ENROLLMENTS

Kg – Grade 1
457 ÷ 451 = 1.01

Grade 2 – 3
516 ÷ 499 = 1.03

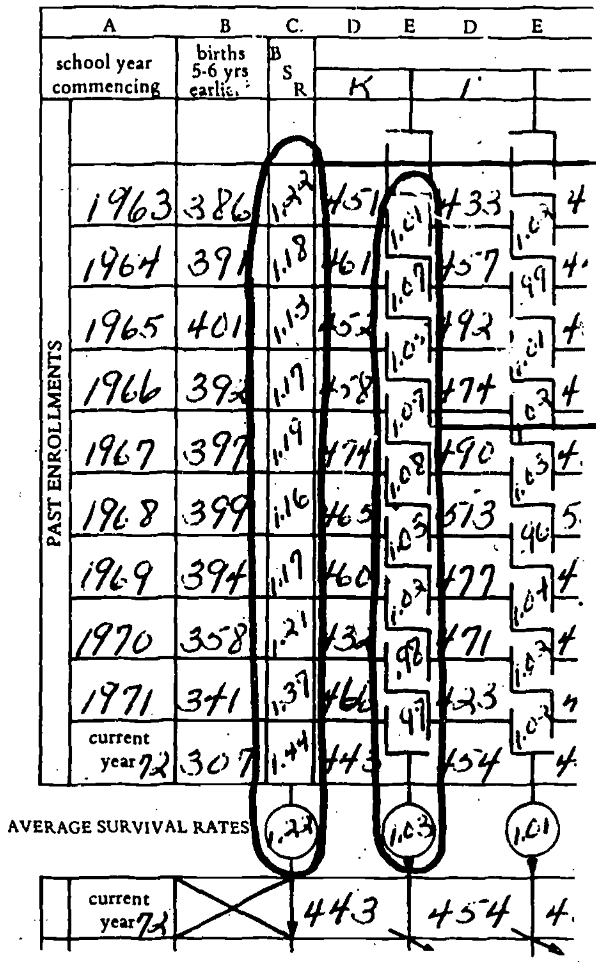
STEP 5

<u>Check Each Column of Survival Rates</u> (Columns C & E's) for unusual deviations. Irregularities may be due to faulty data or transfer errors. They may also point up changes in district boundaries or in the definition of statistics that have previously gone undetected. If found, correct the errors and proceed. If irregular patterns are noted and no errors are found, consult the following section on Modifications. If the rates in each column appear to be randomly distributed with no unusual deviations, proceed with the next step.

STEP 6

<u>Calculate the Average Survival Rate</u> for each column and record the results in the circles corresponding to Columns C and the E's. (Note that the <u>number</u> of individual <u>Birth</u> Survival Rates may differ from the number of <u>Grade</u> Survival Rates.)

Example *Step 6* *Average Survival Rates*



Average Birth Survival Rate

$$\frac{1.22 + 1.18 \ldots + 1.37 + 1.44}{10} = 1.22$$

Average Grade Survival Rate – K-1

$$\frac{1.01 + 1.07 \ldots + .98 + .97}{9} = 1.03$$

STEP 7

Record the Basic Data for Projections. In the first row of the bottom half of Worksheet 2 record the current enrollments for each grade. (These data duplicate the entries in the bottom row of the upper half of the worksheet.) Birth data in Column B has previously been recorded in Step 2-c.

STEP 8

<u>Compute Enrollment Projections</u> by multiplying the number representing each cohort by its corresponding Average Survival Rate, repeating as necessary to complete the desired projections. In terms of Worksheet 2:

a. *Multiply each number of births in Column B by the Average Birth Survival Rate in the circle under Column C. Record the result in the adjacent Column D in the same row.*

b. *Multiply each number in each Column D sequentially by the Average Survival Rate in the accompanying circle under Column E. Following the arrows, record the result in the next column, next row below.*

Example *Step 8* *Enrollment Projections*

A	B	C	D	E	D	E	D
school year commencing	births 5-6 yrs earlier*	BSR	age K		1		2
1963	386	1.22	451	1.01	433	1.07	452
1964	391	1.17	461		457		441

current year 72	307	1.44	443	.97	454	1.03	433
AVERAGE SURVIVAL RATES		1.22		1.03		1.01	
current year 72			443		454		433
1973	327		399		456		
1974	290		354		411		
1975	287		350		365		
1976	275		336		361		
1977	252		307		346		
1978					316		

FUTURE ENROLLMENTS

327 x 1.22 = 399

350 x 1.03 = 361

STEP 9 (Optional)

Record estimated future Resident Births as desired in Column A. Proceed as in Step 8.

The bottom half of Worksheet 2 now represents the projected enrollments for each grade for the next several years.

VII. MODIFICATIONS

Straightforward application of the cohort survival method of enrollment projection will provide good results in normally stable communities, but there are times when modifications are called for. However, since modifications tend to nullify the basic assumptions of the technique, they should be made only when clearly indicated and where the planner's knowledge of the community justifies the change.

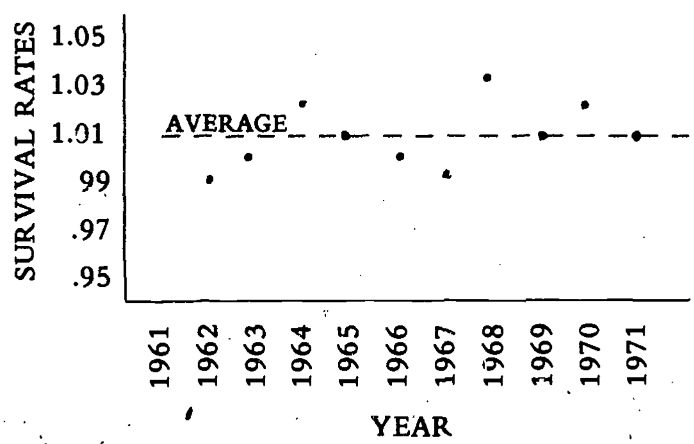
Analyses of survival rates by columns are useful in establishing the wisdom of modifications. A column of survival rates typical of a stable community is illustrated in Figure 1 which shows survival rates from sixth to seventh grades over a period of years. In this instance, the ten rates do not vary more than a few percentage points on either side of the average rate of 1.01. They appear to fall at random about the average and, if plotted on a graph (survival rate versus year), would tend to define a horizontal straight line. In a stable community, the planner may expect similar relationships among other columns of survival rates.

Figure 1

SURVIVAL RATES

Grades 6-7

Year	Rates
1962	.99
1963	1.00
1964	1.02
1965	1.01
1966	1.00
1967	.99
1968	1.03
1969	1.01
1970	1.02
1971	1.01
Average	1.01



Although it is unusual, even in a stable community, for a column of survival rates to vary from its average by as little as three or four percentage points, the random distribution of rates is common. When such a random pattern exists, no modification in the standard procedures is justified. When justified, most modifications are applied to the Average Survival Rates and may affect one, several or all the rates. Following are examples of situations where modifications are justified.

1. *A Permanent Change in Level, Single Column*

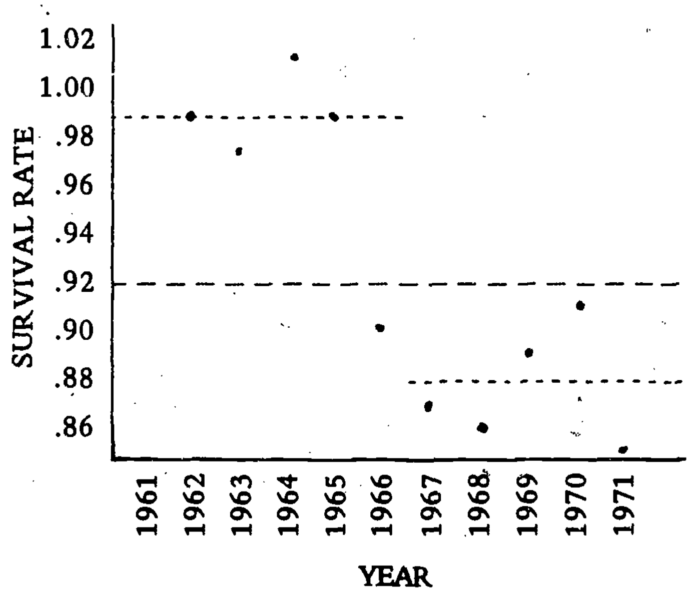
   If a column of survival rates exhibits an abrupt change from one level at the beginning to another at the end, it may mark the opening or closing of an alternate school. For example, if a new regional vocational school serving students in Grades 11 to 14 were opened, attracting 11 percent of the students completing Grade 10, a consequent drop of 11 percentage points could be expected in the local district survival rate from Grades 10 to 11 which would continue in subsequent years. (Figure 2 illustrates such a situation.) On the other hand, if a parochial school were closed and all students completing Grade 6, for example, transferred to public schools, a change in public school survival rates from Grades 6 to 7 in reverse of that in Figure 2 might be expected. When such changes are expected to be permanent, only survival rates subsequent to the change are used in computation of the average. (In Figure 2, one would use only those subsequent to 1965.)

Figure 2

SURVIVAL RATES

Grades 10-11

Year	Rates	
1962	.99	
1963	.97	
1964	1.01	.99
1965	.99	
1966	.90	
1967	.87	
1968	.86	
1969	.89	.88
1970	.91	
1971	.85	
Average	.92	



2. *A Permanent Change in Level, Adjacent Columns*

Changes in survival rate patterns in adjacent columns and in counteracting amounts may reflect a change in promotional policy, as illustrated in Figure 3 which shows the effect of introducing transition classes between kindergarten and Grade 1 in 1966. In this instance, the transition class is considered part of Grade 1 and so increases the Grade 1 survival rate by about 20 percent. Again, if the change is permanent, it would be appropriate to modify computation of the average survival rates for the affected columns in the same manner as in situation 1 above.

Figure 3

TABLE OF ENROLLMENTS AND SURVIVAL RATES SHOWING EFFECT OF INITIATING A TRANSITION CLASS (K to 1 in 1966)

School Year	K Enroll.	K-1 Surv. Rate	1 Enroll.	1-2 Surv. Rate	2 Enroll.	2-3 Surv. Rate	3 Enroll.
1964-5	50		45		48		40
		98		102		98	
1965-6	54		49		46		47
		95		98		96	
1966-7	49		51		48		44
		120		80		97	
1967-8	53		59		41		47
		117		85		100	
1968-9	55		62		50		41
		121		81		99	
1969-70	52		66		50		50
		118		83		98	
			61		54		49

3. *A Permanent Change in Level, All Columns*

Similar changes in level in all columns at the same time suggest an abrupt change in migration patterns in the community. However, an abrupt migration change seldom exhibits as abrupt a change in survival rates as a change caused by such phenomena as new school openings or shifts in promotional policy. If a change in migration patterns is confirmed, the determination of an appropriate adjustment then hinges on how permanent the change is expected to be. Only a thorough knowledge of migration patterns and future plans affecting the district can help the planner with this decision.

4. *A Temporary Change, One or More Columns*

A temporary irregularity in an otherwise consistent pattern could have several causes. The closing of a private secondary school (Grades 7-9 as illustrated in Figure 4 for 1968) with a transfer to the public school of all students at one time, will create a temporary interruption of a stable pattern for the Survival Rates between Grades 7 and 8 and Grades 8 and 9 as well as a permanent one (situation 1) between Grades 6 and 7. An increase in tuition in a nearby independent day school could precipitate similar results, and a temporary change, involving one or several years, might result from an interruption in migratory patterns with the advent of a new housing development (generally affecting all columns at the same time). When such temporary changes have occurred, and there is little expectation of recurrence, it is generally wise to eliminate the irregular rates in computing Average Survival Rates, i.e., eliminate 1.04 and 1.05, each from the groups of rates in its column. Note that the 6-7 grade ratio before 1968 would be eliminated as per example 1 above.

Figure 4

SURVIVAL RATES, GRADES 6-10, SHOWING EFFECT ON PUBLIC SCHOOL ENROLLMENT OF CLOSING NONPUBLIC SCHOOL, GRADES 7-9

Grades ↔ Years	6-7	7-8	8-9	9-10
1965-6	.90	.99	.98	.97
1966-7	.92	.97	.99	.98
1967-8	.91	.98	.97	.96
1968-9	.98	1.04	1.05	.97
1969-70	.97	.99	.97	.96
1970-1	.99	.97	.98	.98
1971-2	.98	.98	.99	.99

5. *A Trend is Established*

A continuous increase or decrease in survival rates, usually evident in all columns at once, indicates growth or decline of the district at accelerated rates. Survival rates in a community undergoing such a decline are illustrated graphically in Figure 5. Should the planner encounter such a phenomenon, he must decide whether he will determine the Average Survival Rate by the standard method, by modifying the "average" to follow the trend, or by some other modification. The arithmetic average of the column may be more appropriate for projection purposes in this instance than one derived by extending the trend, particularly since trends have a habit of reversing themselves.

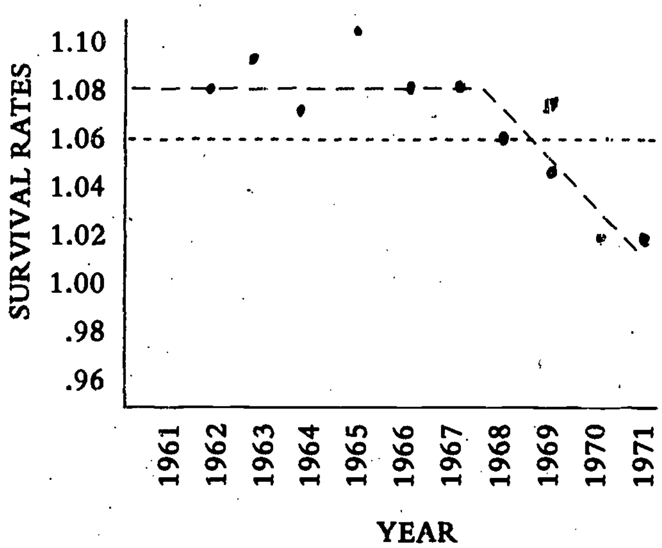
In dealing with an established trend some forecasters consider survival rates of recent years better indicators of the future than those derived from a large amount of historical data; some prefer to work with weighted averages of recent years, i.e., using data from the Step 6 example, Section 6 (1 x 1.02 + 2 x .98 + 3 x .97) ÷ 6 = .98. Although variations of approach may be justified on occasion, they should be used with caution and preferably only by those with considerable experience in forecasting and an extensive knowledge of their district involved.

Figure 5

SURVIVAL RATES

Grades 1-2

Year	Rates
1962	1.08
1963	1.09
1964	1.07
1965	1.10
1966	1.08
1967	1.08
1968	1.06
1969	1.04
1970	1.01
1971	1.01
Average	1.06



6. *Looking Ahead*

Beyond modifications indicated by analysis of survival rates is the frequent necessity to take into account future changes likely to have a lasting effect and a predictable impact on enrollments. If, for example, a new regional-technical school were under construction, it would be unreasonable to ignore it. An adjustment in the computed average survival rate for the appropriate grade should be considered for the year the new school opens and thereafter.

Another kind of adjustment is indicated by the fact that nonpublic schools tend to enroll a constant number rather than a constant proportion of students. Thus, districts with a high proportion of nonpublic school students should use another kind of modification. One way would be to project public and nonpublic enrollments together and then deduct the normal enrollments in nonpublic schools.

It is of course impossible to consider all of the many situations that might preclude strict application of the cohort-survival model. It must suffice to note some of them and to suggest some of the modifications that experienced planners have employed successfully. It must then be left to the planner to use his judgment in the application of processes and calculations, to question his rationale at every step, and to make such modifications in standard procedures as may be fully justified.

VIII. CONFIDENCE LIMITS

The procedure for calculating confidence limits is presented in two major parts:

1. *Calculation of Confidence Limits for Each Individual Enrollment Estimate*

   The determination of confidence limits for individual projections involves two sets of computations: one for the initial year of schooling based on Birth Survival Rates; the other, for all other years of schooling based on all other survival rates (Grades Survival Rates or Age Survival Rates, depending on the statistics used). Although the computations are very similar, the number of Birth Survival Rates employed in the computations is much smaller than the number of Grade or Age Survival Rates. (Using ten years of original enrollment data for grades kindergarten through twelve, for example, one would normally employ eight or more Birth Survival Rates and 8 rows of 12 columns, or 96 Grade Survival Rates in the computations.)

2. *Calculation of Confidence Limits for Groups of Estimates (primary grades, middle school years, high school, total school years)*

   Confidence limits for enrollment projections are more often useful in terms of totals (i.e., elementary or high school enrollments) than in terms of individual projections. To determine the confidence interval for a group of individual projections—primary Grades 1-3, for example—an interval for each of the individual projections must be calculated. These data may then be used in accordance with directions under *Calculating Confidence Limits for Groups of Estimates* to obtain the limits for the total group.

A confidence interval in enrollment forecasting may be considered as a range of enrollments within which a given forecasted enrollment may be expected to fall (due to chance variation of influential factors involved) at a chosen level of confidence. Provision is made in this workbook for a choice of any one of four levels. Tables are included for the computation of confidence limits to match confidence levels of 50%, 80%, 90%, and 95%. As a general rule, the 80% level is considered most practical.

The accompanying worksheets and tables have been designed to find confidence limits on an age or grade basis. The planner may use whichever is most appropriate for the school district under consideration. Although specific directions are given for grade projections, the directions are correct if "age" is substituted for "grade" wherever it appears.

CALCULATING CONFIDENCE LIMITS FOR INDIVIDUAL ENROLLMENT ESTIMATES.

STEP 1

Calculate Relative Birth Survival Rates - c(b).

a. *Transfer Birth Survival Rates from Worksheet 2, Column C, to Column a(b) of Worksheet 3, eliminating the rate for the earliest year.**

b. *Transfer the average Birth Survival Rate from the circle in Column C of Worksheet 2 to the circle in Column b(b) of Worksheet 3.*

c. *Divide each Birth Survival Rate in Column a(b) by the Average Birth Survival Rate in Column b(b) and record the results as the Relative Birth/Survival Rates in Column c(b).*

STEP 2

Calculate Relative Survival Rates - c.

a. *Transfer Survival Rates from Columns E of Worksheet 2 to Columns a of Worksheet 3, eliminating the rates for the earliest year.**

b. *Transfer the Average Survival Rates from the circles in Columns E of Worksheet 2 to the circles in Column b of Worksheet 3.*

c. *Divide the Survival Rates in Columns a by the Average Survival Rates in Columns b and record the results as the Relative Survival Rates in Columns c.*

Example *Steps 1 and 2*

WORKSHEET NO. 3 – RELATI

BIRTHS TO GRADE/AGE K			GRADE/AGE K TO I			GRADE/AGE I TO II		
a(b)	b(b)	c(b)	a	b	c	a	b	c
1.18	1.22	.97	1.07	1.03	1.04	.99	1.01	.98
[illegible]		.97	1.05		1.02	1.01		1.00
1.17	Average Survival Rate	.96	1.07	Average Survival Rate	1.04	1.02	Average Survival Rate	1.01
1.19		.98	1.08		1.05	1.03		1.02
1.16		.95	1.03		1.00	.96		.95
1.17		.96	1.02		.97	1.04		1.03
1.21		.99	.98		.95	1.02		1.01
1.37		1.12	.97		.94	1.02		1.01
1.44		1.18						

Relative Birth Survival Rates

$a(b)/b(b) = c(b)$

$1.18/1.22 = .97$

Relative Survival Rates

$a/b = c$

$1.04/1.01 = 1.03$

*This is a statistical procedure to eliminate dependence between the rates and their average.

STEP 3

Determine the Value of V(B).

a. *Pick the largest value of c(b) calculated in Step 1. Let it be $C_2(B)$.**

b. *Pick the smallest value of c(b) calculated in Step 1. Let it be $C_1(B)$.*

c. *Calculate W(B).*

$$W(B) = \frac{C_2(B)}{C_1(B)}$$

d. *Consult Table 1 to find the Value of V corresponding to the value of W(B). (W in Table 1)*
*In Table 1, N represents the number of years of enrollment data employed in calculating the original survival rates in Worksheet 2. (In the example included herein, N = 10.)***

e. *Multiply V as determined from Table 1 by 1.33. The result is V(B).*

STEP 4

Determine the Value of V.

a. *Count the number of Relative Survival Rates recorded in Step 3. Let the number be S.*

b. *Multiply S by .07 (7%) and round to the next highest digit. Let the digit be D.*

c. *From the entire number (S) of Relative Survival Rates, suppose they are placed in rank order from largest to smallest. Find the Rate which is D places from the largest Rate (counting the largest as one).*
Let this Rate be C_2.
Find the Rate which is D places from the smallest Rate (counting the smallest as one).
Let this Rate be C_1.

d. *Calculate W.*

$$W = \frac{C_2}{C_1}$$

---

*Statistical procedures call for the 7th and 93rd percentile values. This procedure is accurate for any number of Relative Birth Survival Rates less than 15.

**If modification in average survival rates, based on decisions to use only a portion of the computed survival rates in determining the average, have been made in the projection process, it may be argued that matching modifications in N (as applied in Table 1) would be appropriate. However, NESDEC takes the position that, regardless of the modifications, the confidence limits should be computed from the full range of historical data originally considered for projection purposes (all that entered in Worksheet 2).

e. *Consult Table 1 to find the values of V corresponding to the value of W.*

*In Table 1, N represents the number of years of enrollment data employed in calculating the original survival rates in Worksheet 2.**

Example *Steps 3 and 4*

The Value of V(B)

$$\frac{\boxed{1.18}\ C_2(B)}{\boxed{.93}\ C_1(B)} = W(B)\quad \boxed{1.269}$$

$\boxed{.008}$ $V \times 1.33 = V(B)$ $\boxed{.011}$

(from Table 1)

The Value of V

RELATIVE SURVIVAL RATES IN RANK ORDER

HIGHEST	1	1.07
	2	1.06
	3	1.05
	4	1.05
	5	1.05
	6	1.05
	7	1.04
	8	1.04
LOWEST	8	.97
	7	.97
	6	.97
	5	.96
	4	.95
	3	.95
	2	.94
	1	.91

$$\frac{\boxed{1.04}\ C_2}{\boxed{.97}\ C_1} \doteq W \quad \boxed{1.072}$$

$V = \boxed{.001}$

(from Table 1)

(Data from .96 Relative Survival Rates)

*If modification in average survival rates, based on decisions to use only a portion of the computed survival rates in determining the average, have been made in the projection process, it may be argued that matching modifications in N (as applied in Table 1) would be appropriate. However, NESDEC takes the position that regardless of the modifications, the confidence limits should be computed from the full range of historical data originally considered for projection purposes (all that entered in Worksheet 2).

STEP 5

Determine the Confidence Intervals.

a. *From Worksheet 3, transfer the appropriate values of V(B) and V to Worksheet 4 in the spaces provided. (For the early grades, those whose enrollments are dependent on birth to initial grade survival, the value of V(B) will be used in the calculations in addition to V.)*

b. *Select the level of confidence acceptable for projection: 50%, 80%, 90%, 95% (the higher the percent, the larger are the confidence intervals). Record the choice in the space provided in Worksheet 4.*

c. *Determine the value of K for each projected year and grade. Note the value of M in Column 2, Worksheet 4 for the appropriate grade. N is the same number used in Step 4e (the number of years of enrollment data employed in calculating the original survival rates in Worksheet 2). From Table 2 for the chosen level of confidence and the appropriate values of M and N, find the value of K. Record in Column 3. If M is 0, use 0 for K also. Repeat for each year and each grade.*

d. *Determine the value of J for each projected year and grade.*

(1) *If M is not starred (*), multiply the value of K in Column 3 by the value of V. (Four decimal places) Enter in Column 4.* $J = K \times V$ *if M is unstarred.*

(2) *If M is starred (*)*

(a) *Multiply the value of K corresponding to N, and M = 1 as determined from Table 2 for the chosen level of confidence by the value of V(B).*

(b) *Multiply the value of K in Column 3 by V.*

(c) *Add the results of (a) and (b) above. Record the result in Column 4. (Four decimal places)* $J = K_{1,N} \times V(B) + K \times V$ *if M is starred.*

e. *Determine the value of R for each projected year and grade. From Table 3 and the values of J determined in Step 5d above, find the corresponding values of R. (It is suggested that one enter the table with J, as determined in Step 5d, rounded in the fourth decimal place to the nearest multiple of 5. In other words, it is not necessary to interpolate.) Record the value of R in Column 5.*

f. *Calculate the Confidence Intervals for each projected year and grade.*

(1) *From Worksheet 2 record the projected enrollment corresponding to the appropriate year and grade in Column 7.*

(2) *Divide the projected enrollment (E) in Column 7 by the value of R in Column 5 to find the lower limit of the confidence*

Example *Step 5*

WORKSHEET NO. 4.1 **Confidence Level** 80 % V(B) = .011, V = .001

INITIAL SCHOOL YEAR K

1	2	3	4	5	6	7	8	9	10
YEAR	M	K	J	R	ℓ	E	u	Δℓ	Δu
73	0*	0	.0199	1.1519	346	399	460		
74	0*	0	.0199	1.1519	307	354	408		
75	0*	0	.0199		304	350	403		
76	0*	0	.0199		292	336	387		
77	0*	0	.0199		267	307	354		
	0*								

SECOND SCHOOL YEAR I

1	2	3	4	5	6	7	8	9	10
YEAR	M	K	J	R	ℓ	E	u	Δℓ	Δu
73	1	1.808	.0018	1.0457	436	456	477		
74	1*	1.808	.0216	1.1579	355	411	476		
75	1*	1.808	.0216	1.1579	315	365	423		
76	1*	1.808	.0216		312	361	418		
77	1*	1.808	.0216		299	346	401		
78	1*	1.808	.0216		273	316	366		
	1*								

*The INITIAL SCHOOL YEAR may be Grade 1, Kindergarten; Age 6 or 5, depending on projection

THIRD SCHOOL YEAR II

1	2	3	4	5	6	7	8	9	10
YEAR	M	K	J	R	ℓ	E	u	Δℓ	Δu
73	1	1.808	.0018	1.0457	439	459	480		
74	2	1.972	.0020	1.0457	441	461	482		
75	2*	1.972	.0219	1.1599	358	415	481		
76	2*			1.1599	318	369	428		
77	2*				315	365	423		
78	2*				301	349	405		
79	2*				275	319	370		
	2*								

FOURTH SCHOOL YEAR III

1	2	3	4	5	6	7	8	9	10
YEAR	M	K	J	R	ℓ	E	u	Δℓ	Δu
73	1	1.808	.0018	1.0457	423	442	462		
74	2	1.972	.0020	1.0457	448	468	489		
75	3	2.137	.0021	1.0457	449	470	491		
76	3*	2.137	.0220	1.1599	365	423	491		
77	3*			1.1599	324	376	436		
78	3*				321	372	431		
79	3*				307	356	413		
80	3*				280	325	377		
	3*								

interval (ℓ). Record in Column 6.

(3) *Multiply the projected enrollment (E) in Column 7 by the value of R in Column 5 to find the upper limit of the confidence interval (u). Record in Column 8.*

CALCULATING CONFIDENCE LIMITS FOR GROUPS OF ESTIMATES

Confidence limits for groups of estimates (elementary grades, secondary grades, totals) are based on individual confidence limit computations, and determined as follows:

STEP 1

Determine the deviation ($\Delta \ell$) of each lower confidence limit from its projected enrollment (E). In terms of Worksheet 4:

Subtract each value of ℓ (Column 6) from the corresponding value of E (Column 7) and record the result in Column 9.

STEP 2

Determine the deviation (Δu) of each upper confidence limit from its projected enrollment (E). In terms of Worksheet 4:

Subtract each value of E (Column 7) from the corresponding value of u (Column 8) and record the result in Column 10.

Example *Steps 1 and 2*

II

5	6	7	8	9	10
R	ℓ	E	u	$\Delta \ell$	Δu
1.0457	439	459	480	20	21
1.0457	441	461	482	20	21
1.1599	358	415	481	57	66
1.1599	318	369	428	51	59
	315	365	423	50	58
	301	349	405	48	56
	275	319	370	44	51

$480 - 459 = 21$

$365 - 315 = 50$

STEP 3

Transfer pertinent data to Worksheet 5.

a. *In Column 1 enter the years for which confidence limits are desired (i.e., 1973, 1975, 1977, 1979).*

b. *For each year chosen, enter the projected enrollments in Row a for that group of grades for which confidence limits are desired (i.e., Grades K through 8, or 9, 10, 11, 12. Worksheet 5 is made to handle one group of grades at a time.)*

c. *For the years and grades chosen, find the corresponding Δu in Column 10 of Worksheet 4, square the number and record the result in Row b.*

d. *For the years and grades chosen, find the corresponding $\Delta \ell$ in Column 9 of Worksheet 4, square the number and record the result in Row c.*

STEP 4

Determine the sum of each of the three statistics recorded for the chosen group of grades. In terms of Worksheet 5:

a. *Add the numbers in each Row a. Enter the sum in Column 16 in the same Row a. (This number is the projected enrollment for the total of the group of grades chosen in Step 3b above.)*

b. *Add the squares of the upper limit deviations ($\overline{\Delta u}^2$). Record each sum in the proper row in Column 16.*

c. *Add the squares of the lower limit deviations ($\overline{\Delta \ell}^2$). Record each sum in the proper row of Column 16.*

STEP 5

Determine the square root of the sum of the squares of the deviations. In terms of Worksheet 5:

a. *Compute the square root of each sum ($\Sigma\ \overline{\Delta u}^2$) found in Column 16. Record it in Row b in Column 17.*

b. *Compute the square root of each sum ($\Sigma\ \overline{\Delta \ell}^2$) found in Column 16. Record it in Row c in Column 17.*

STEP 6

Determine the confidence limits for each of the years and for the groups chosen.

a. *Add each number ($\sqrt{\Sigma\ \overline{\Delta u}^2}$) in Column 17, Row b, to the projected group enrollment (ΣE) in Row a of Column 16 to find the group upper confidence limit (u). Record the result in the box marked u in Column 18.*

b. *Subtract each number ($\sqrt{\Sigma\ \overline{\Delta \ell}^2}$) in Column 17, Row c, from the projected group enrollment (ΣE) in Row a of Column 16 to find*

Example *Steps 3, 4 and 5*

WORKSHEET NO. 5 **CONFIDENCE LIMITS FOR GROUPS – TOTALS**

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
YEAR	STATIS-TIC	INITIAL SCHOOL YEAR	GRADE or AGE												Σ	$\sqrt{\Sigma\Delta^2}$	CONF. LIMITS
			I	II	III	IV	V	VI									
a	E	399	456	459	442	480	438	505							3229		
73 b	$\overline{\Delta u}^2$	3721	441	441	400	484	484	529							6500	81	u 3310
c	$\overline{\Delta \ell}^2$	2809	400	400	361	441	441	484							5336	73	ℓ 3156
a	E	354	411	461	489	446	475	493							3129		
74 b	$\overline{\Delta u}^2$	2916	4225	441	441	400	484	529							9436	97	u 3226
c	$\overline{\Delta \ell}^2$	2209	3136	400	400	361	441	484							7431	86	ℓ 3043
a	E	350	365	415	491	473	442	480							3016		
75 b	$\overline{\Delta u}^2$	2809	3364	4356	441	484	400	484							12338	111	u 3127
c	$\overline{\Delta \ell}^2$	2116	2500	3249	441	441	361	441							9549	98	ℓ 2918
a	E	336	361	369	491	475	468	446							2946		
76 b	$\overline{\Delta u}^2$	2601	3249	3481	4624	576	576	529							15636	127	u 3073
c	$\overline{\Delta \ell}^2$	1936	2401	2601	3364	529	529	484							11844	109	ℓ 2837
a	E	307	346	365	436	427	470	473							2824		
77 b	$\overline{\Delta u}^2$	2209	3025	3364	3600	4624	576	576							17974	134	u 2958
c	$\overline{\Delta \ell}^2$	1600	2209	2500	2704	3481	529	529							13552	118	ℓ 2706
a	E		316	349	431	380	423	475									
78 b	$\overline{\Delta u}^2$		2500	3136	3481	3721	4624	576									u
c	$\overline{\Delta \ell}^2$		1849	2304	2601	2704	3481	529									ℓ
a	E			319	413	376	376	437									
79 b	$\overline{\Delta u}^2$			2601	3249	3600	3721	4761									u
c	$\overline{\Delta \ell}^2$			1936	2401	2704	2704	3481									ℓ
a	E				377	360	372	380									
80 b	$\overline{\Delta u}^2$				2704	3364	3600	3721									u
c	$\overline{\Delta \ell}^2$				2025	2500	2704	2809									ℓ
a	E					328	356	376									
81 b	$\overline{\Delta u}^2$					2704	3364	3721									u
c	$\overline{\Delta \ell}^2$					2025	2500	2704									ℓ
a	E																
b	$\overline{\Delta u}^2$																u
c	$\overline{\Delta \ell}^2$																ℓ

the group lower confidence limit (ℓ). Record the result in the box marked ℓ in Column 18.

STEP 7

Repeat Steps 3 through 7 for other groups of grades for which confidence limits are desired.

TABLE I

VALUES OF V

W \ N	3	4	5	6	7	8	9	10	11	12	13	14	15
1.00	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
1.01	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001
1.02	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001
1.03	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001
1.04	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001
1.05	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001
1.06	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001
1.07	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001
1.08	.002	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001
1.09	.002	.002	.002	.002	.001	.001	.001	.001	.001	.001	.001	.001	.001
1.10	.002	.002	.002	.002	.002	.002	.002	.002	.002	.002	.002	.002	.002
1.11	.002	.002	.002	.002	.002	.002	.002	.002	.002	.002	.002	.002	.002
1.12	.003	.002	.002	.002	.002	.002	.002	.002	.002	.002	.002	.002	.002
1.13	.003	.003	.003	.003	.002	.002	.002	.002	.002	.002	.002	.002	.002
1.14	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003
1.15	.004	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003
1.16	.004	.004	.004	.004	.003	.003	.003	.003	.003	.003	.003	.003	.003
1.17	.005	.004	.004	.004	.004	.004	.004	.004	.004	.004	.004	.004	.004
1.18	.005	.005	.004	.004	.004	.004	.004	.004	.004	.004	.004	.004	.004
1.19	.006	.005	.005	.005	.005	.004	.004	.004	.004	.004	.004	.004	.004
1.20	.006	.006	.005	.005	.005	.005	.005	.005	.005	.005	.005	.005	.005
1.21	.007	.006	.006	.006	.005	.005	.005	.005	.005	.005	.005	.005	.005
1.22	.007	.007	.006	.006	.006	.006	.006	.006	.005	.005	.005	.005	.005
1.23	.008	.007	.007	.006	.006	.006	.006	.006	.006	.006	.006	.006	.006
1.24	.008	.008	.007	.007	.007	.007	.006	.006	.006	.006	.006	.006	.006
1.25	.009	.008	.008	.007	.007	.007	.007	.007	.007	.007	.007	.007	.007

43

TABLE 1 (Cont'd.)

VALUES OF V

W \ N	3	4	5	6	7	8	9	10	11	12	13	14	15
1.26	.010	.009	.008	.008	.008	.008	.007	.007	.007	.007	.007	.007	.007
1.27	.010	.009	.009	.008	.008	.008	.008	.008	.008	.008	.008	.008	.008
1.28	.011	.010	.009	.009	.009	.008	.008	.008	.008	.008	.008	.008	.008
1.29	.012	.010	.010	.009	.009	.009	.009	.009	.009	.009	.009	.009	.008
1.30	.012	.011	.010	.010	.010	.010	.009	.009	.009	.009	.009	.009	.009
1.31	.013	.012	.011	.011	.010	.010	.010	.010	.010	.010	.010	.010	.009
1.32	.014	.012	.012	.011	.011	.011	.010	.010	.010	.010	.010	.010	.010
1.33	.014	.013	.012	.012	.011	.011	.011	.011	.011	.011	.011	.011	.010
1.34	.015	.014	.013	.012	.012	.012	.012	.011	.011	.011	.011	.011	.011
1.35	.016	.014	.013	.013	.013	.012	.012	.012	.012	.012	.012	.012	.012
1.36	.017	.015	.014	.014	.013	.013	.013	.013	.012	.012	.012	.012	.012
1.37	.018	.016	.015	.014	.014	.013	.013	.013	.013	.013	.013	.013	.013
1.38	.018	.016	.015	.015	.014	.014	.014	.014	.014	.013	.013	.013	.013
1.39	.019	.017	.016	.015	.015	.015	.014	.014	.014	.014	.014	.014	.014
1.40	.020	.018	.017	.016	.016	.015	.015	.015	.015	.015	.015	.014	.014
1.41	.021	.019	.017	.017	.016	.016	.016	.016	.015	.015	.015	.015	.015
1.42	.022	.019	.018	.017	.017	.017	.016	.016	.016	.016	.016	.016	.016
1.43	.023	.020	.019	.018	.018	.017	.017	.017	.017	.017	.016	.016	.016
1.44	.023	.021	.020	.019	.018	.018	.018	.017	.017	.017	.017	.017	.017
1.45	.024	.022	.020	.020	.019	.019	.018	.018	.018	.018	.018	.018	.017
1.46	.025	.022	.021	.020	.020	.019	.019	.019	.019	.018	.018	.018	.018
1.47	.026	.023	.022	.021	.020	.020	.020	.019	.019	.019	.019	.019	.019
1.48	.027	.024	.023	.022	.021	.021	.020	.020	.020	.020	.020	.019	.019
1.49	.028	.025	.023	.022	.022	.021	.021	.021	.021	.020	.020	.020	.020
1.50	.029	.026	.024	.023	.023	.022	.022	.021	.021	.021	.021	.021	.021

TABLE 2A

VALUES OF K
95% Confidence Interval

N \ M	1	2	3	4	5	6	7	8	9	10	11
3	5.122	6.403	7.683	8.964	10.244	11.525	12.805	14.086	15.366	16.647	17.927
4	4.802	5.762	6.723	7.683	8.644	9.604	10.564	11.525	12.485	13.446	14.406
5	4.610	5.378	6.147	6.915	7.683	8.452	9.220	9.988	10.756	11.525	12.293
6	4.482	5.122	5.762	6.403	7.043	7.683	8.323	8.964	9.604	10.244	10.885
7	4.390	4.939	5.488	6.037	6.586	7.134	7.683	8.232	8.781	9.330	9.878
8	4.322	4.802	5.282	5.762	6.243	6.723	7.203	7.683	8.163	8.644	9.124
9	4.268	4.695	5.122	5.549	5.976	6.403	6.830	7.256	7.683	8.110	8.537
10	4.226	4.610	4.994	5.378	5.762	6.147	6.531	6.915	7.299	7.683	8.067
11	4.191	4.540	4.889	5.239	5.588	5.937	6.286	6.635	6.985	7.334	7.683
12	4.162	4.482	4.802	5.122	5.442	5.762	6.083	6.403	6.723	7.043	7.363
13	4.137	4.433	4.728	5.024	5.319	6.615	5.910	6.206	6.501	6.797	7.092
14	4.116	4.390	4.665	4.939	5.214	5.488	5.762	6.037	6.311	6.586	6.860
15	4.098	4.354	4.610	4.866	5.122	5.378	5.634	5.890	6.147	6.403	6.659

TABLE 2B

VALUES OF K
90% Confidence Interval

N \ M	1	2	3	4	5	6	7	8	9	10	11
3	3.608	4.510	5.412	6.314	7.216	8.118	9.020	9.922	10.824	11.726	12.628
4	3.383	4.059	4.736	5.412	6.089	6.765	7.442	8.118	8.795	9.471	10.148
5	3.247	3.788	4.330	4.871	5.412	5.953	6.494	7.036	7.577	8.118	8.659
6	3.157	3.608	4.059	4.510	4.961	5.412	5.863	6.314	6.765	7.216	7.667
7	3.093	3.479	3.866	4.252	4.639	5.025	5.412	5.799	6.185	6.572	6.958
8	3.044	3.383	3.721	4.059	4.397	4.736	5.074	5.412	5.750	6.089	6.427
9	3.007	3.307	3.608	3.909	4.209	4.510	4.811	5.111	5.412	5.713	6.013
10	2.977	3.247	3.518	3.788	4.059	4.330	4.600	4.871	5.141	5.412	5.683
11	2.952	3.198	3.444	3.690	3.936	4.182	4.428	4.674	4.920	5.166	5.412
12	2.932	3.157	3.383	3.608	3.834	4.059	4.285	4.510	4.736	4.961	5.187
13	2.914	3.122	3.330	3.539	3.747	3.955	4.163	4.371	4.579	4.788	4.996
14	2.899	3.093	3.268	3.479	3.672	3.866	4.059	4.252	4.446	4.639	4.832
15	2.886	3.067	3.247	3.428	3.608	3.788	3.969	4.149	4.330	4.510	4.690

TABLE 2C

VALUES OF K
80% Confidence Interval

N \ M	1	2	3	4	5	6	7	8	9	10	11
3	2.191	2.739	3.287	3.835	4.383	4.931	5.478	6.026	6.574	7.122	7.670
4	2.054	2.465	2.876	3.287	3.698	4.109	4.520	4.931	5.341	5.752	6.163
5	1.972	2.301	2.630	2.958	3.287	3.616	3.944	4.273	4.602	4.931	5.259
6	1.917	2.191	2.465	2.739	3.013	3.287	3.561	3.835	4.109	4.383	4.657
7	1.878	2.113	2.348	2.583	2.817	3.052	3.287	3.522	3.757	3.991	4.226
8	1.849	2.054	2.260	2.465	2.671	2.876	3.082	3.287	3.492	3.698	3.903
9	1.826	2.009	2.191	2.374	2.557	2.739	2.922	3.104	3.287	3.470	3.652
10	1.808	1.972	2.137	2.301	2.465	2.630	2.794	2.958	3.123	3.287	3.451
11	1.793	1.942	2.092	2.241	2.391	2.540	2.689	2.839	2.988	3.138	3.287
12	1.780	1.917	2.054	2.191	2.328	2.465	2.602	2.739	2.876	3.013	3.150
13	1.770	1.896	2.023	2.149	2.276	2.402	2.528	2.655	2.781	2.908	3.034
14	1.761	1.878	1.996	2.113	2.230	2.348	2.465	2.583	2.700	2.817	2.935
15	1.753	1.863	1.972	2.082	2.191	2.301	2.411	2.520	2.630	2.739	2.849

TABLE 2D

VALUES OF K

50% Confidence Interval

N \ M	1	2	3	4	5	6	7	8	9	10	11
3	.606	.757	.909	1.060	1.211	1.363	1.514	1.666	1.817	1.969	2.120
4	.568	.681	.795	.909	1.022	1.136	1.249	1.363	1.476	1.590	1.704
5	.545	.636	.727	.818	.909	.999	1.090	1.181	1.272	1.363	1.454
6	.530	.606	.681	.757	.833	.909	.984	1.060	1.136	1.211	1.287
7	.519	.584	.649	.714	.779	.844	.909	.973	1.038	1.103	1.168
8	.511	.568	.625	.681	.738	.795	.852	.909	.965	1.022	1.079
9	.505	.555	.606	.656	.707	.757	.808	.858	.909	.959	1.010
10	.500	.545	.591	.636	.681	.727	.772	.818	.863	.909	.954
11	.496	.537	.578	.619	.661	.702	.743	.785	.826	.867	.909
12	.492	.530	.568	.606	.644	.681	.719	.757	.795	.833	.871
13	.489	.524	.559	.594	.629	.664	.699	.734	.769	.804	.839
14	.487	.519	.552	.584	.617	.649	.681	.714	.746	.779	.811
15	.485	.515	.545	.575	.606	.636	.666	.697	.727	.757	.787

TABLE 3

VALUES OF R

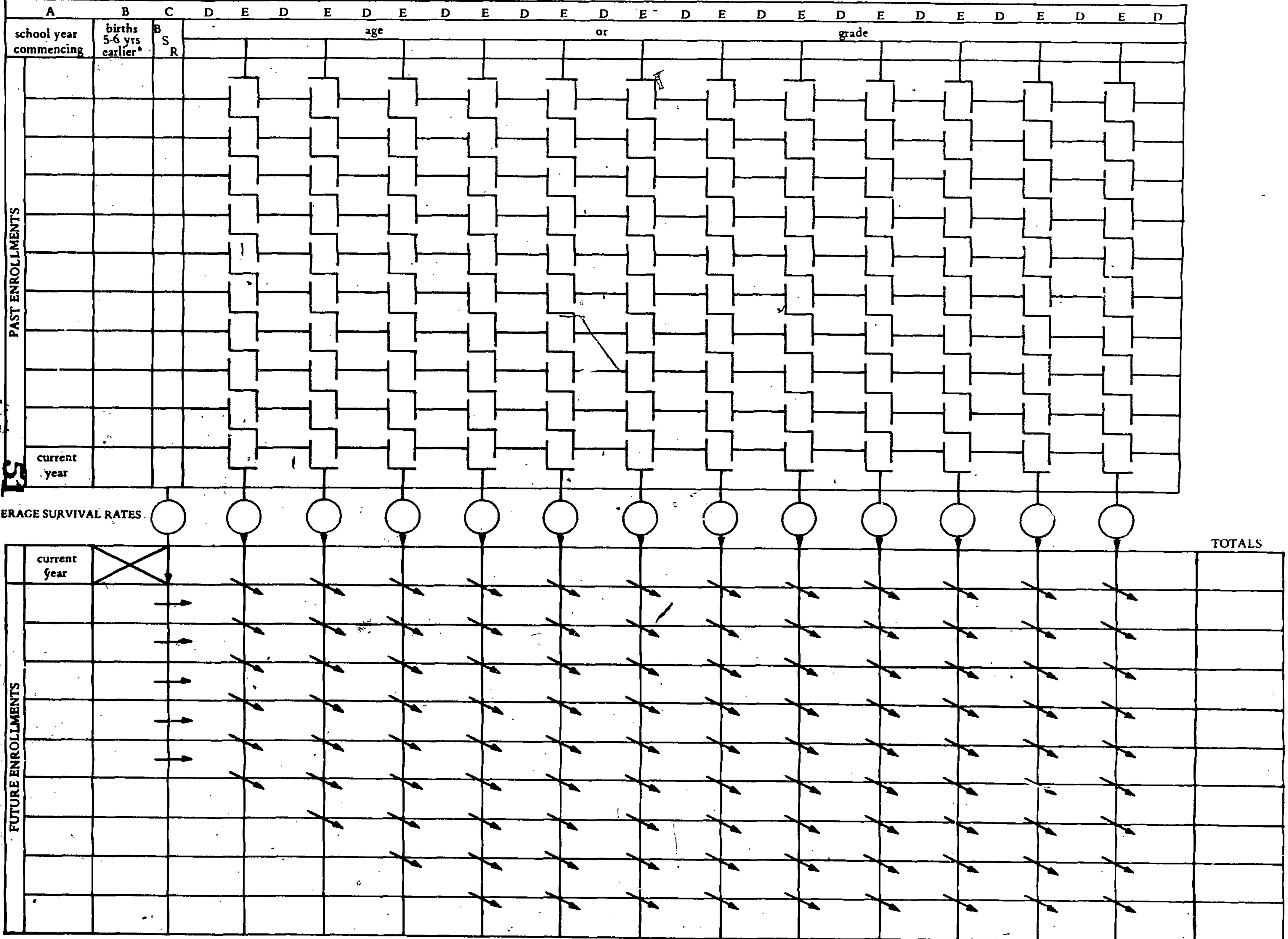
J	R	J	R	J	R
.0005	1.0226	.0170	1.1393	.0335	1.2009
.0010	1.0321	.0175	1.1414	.0340	1.2025
.0015	1.0395	.0180	1.1436	.0345	1.2041
.0020	1.0457	.0185	1.1457	.0350	1.2057
.0025	1.0513	.0190	1.1478	.0355	1.2073
.0030	1.0563	.0195	1.1499	.0360	1.2089
.0035	1.0609	.0200	1.1519	.0365	1.2105
.0040	1.0653	.0205	1.1539	.0370	1.2121
.0045	1.0694	.0210	1.1559	.0375	1.2137
.0050	1.0733	.0215	1.1579	.0380	1.2152
.0055	1.0770	.0220	1.1599	.0385	1.2168
.0060	1.0805	.0225	1.1618	.0390	1.2183
.0065	1.0840	.0230	1.1638	.0395	1.2199
.0070	1.0873	.0235	1.1657	.0400	1.2214
.0075	1.0905	.0240	1.1676	.0405	1.2229
.0080	1.0936	.0245	1.1694	.0410	1.2244
.0085	1.0966	.0250	1.1713	.0415	1.2259
.0090	1.0995	.0255	1.1731	.0420	1.2275
.0095	1.1024	.0260	1.1750	.0425	1.2289
.0100	1.1052	.0265	1.1768	.0430	1.2304
.0105	1.1079	.0270	1.1786	.0435	1.2319
.0110	1.1106	.0275	1.1804	.0440	1.2334
.0115	1.1132	.0280	1.1821	.0445	1.2349
.0120	1.1158	.0285	1.1839	.0450	1.2363
.0125	1.1183	.0290	1.1857	.0455	1.2378
.0130	1.1208	.0295	1.1874	.0460	1.2392
.0135	1.1232	.0300	1.1891	.0465	1.2407
.0140	1.1256	.0305	1.1908	.0470	1.2421
.0145	1.1280	.0310	1.1925	.0475	1.2435
.0150	1.1303	.0315	1.1942	.0480	1.2449
.0155	1.1326	.0320	1.1959	.0485	1.2464
.0160	1.1348	.0325	1.1975	.0490	1.2478
.0165	1.1371	.0330	1.1992	.0495	1.2492
				.0500	1.2506

WORKSHEET NO. 1

year*	births	PAST ENROLLMENTS age or grade (indicate in top row)												
current year														

*Normally calendar year for births and school year (starting in the indicated year) for enrollments

WORKSHEET NO. 2 – PROJECTED ENROLLMENTS



*Births 6 yrs. earlier are used for 1st grade projections. Births 5 yrs. earlier are used for kindergarten projections.

WORKSHEET NO. 3 – RELATIVE SURVIVAL RATES (a/b = c) and THE DETERMINATION OF V

BIRTHS TO GRADE/AGE ___			GRADE/AGE ___ TO ___			GRADE/AGE ___ TO ___			GRADE/AGE ___ TO ___			GRADE/AGE ___ TO ___			GRADE/AGE ___ TO ___			GRADE/AGE ___ TO ___			GRADE/AGE ___ TO ___		
a(b)	b(b)	c(b)	a	b	c	a	b	c	a	b	c	a	b	c	a	b	c	a	b	c	a	b	c
	Average Survival Rate			Average Survival Rate			Average Survival Rate			Average Survival Rate			Average Survival Rate			Average Survival Rate			Average Survival Rate			Average Survival Rate	

GRADE/AGE ___ TO ___			GRADE/AGE ___ TO ___			GRADE/AGE ___ TO ___			GRADE/AGE ___ TO ___			GRADE/AGE ___ TO ___		
a	b	c	a	b	c	a	b	c	a	b	c	a	b	c
	Average Survival Rate			Average Survival Rate			Average Survival Rate			Average Survival Rate			Average Survival Rate	

The Value of V(B)

$$\frac{\boxed{}\ C_2(B)}{\boxed{}\ C_1(B)} = W(B) \quad \boxed{}, \quad \boxed{}\ V \times 1.33 = V(B) \quad \boxed{}$$

(from Table 1)

RELATIVE SURVIVAL RATES in RANK ORDER

HIGHEST	
1	
2	
3	
4	
5	
6	
7	
8	

LOWEST	
8	
7	
6	
5	
4	
3	
2	
1	

The Value of V

$$\frac{\boxed{}\ C_2}{\boxed{}\ C_1} = W \quad \boxed{}$$

V = $\boxed{}$
(from Table 1)

Note: From N years of Enrollment data; there are N-1 Survival Rates and N-2 Relative Survival Rates in each column.

WORKSHEET NO. 4.1 **Confidence Level** ________ **V(B) =** ________ **V =** ________

INITIAL SCHOOL YEAR ________

1	2	3	4	5	6	7	8	9	10
YEAR	M	K	J	R	ℓ	E	u	$\Delta \ell$	Δu
	0*								
	0*								
	0*								
	0*								
	0*								
	0*								

SECOND SCHOOL YEAR ________

1	2	3	4	5	6	7	8	9	10
YEAR	M	K	J	R	ℓ	E	u	$\Delta \ell$	Δu
	1								
	1*								
	1*								
	1*								
	1*								
	1*								
	1*								

The INITIAL SCHOOL YEAR may be Grade 1, Kindergarten; Age 6 or 5, depending on projection

THIRD SCHOOL YEAR ________

1	2	3	4	5	6	7	8	9	10
YEAR	M	K	J	R	ℓ	E	u	$\Delta \ell$	Δu
	1								
	2								
	2*								
	2*								
	2*								
	2*								
	2*								
	2*								

FOURTH SCHOOL YEAR ________

1	2	3	4	5	6	7	8	9	10
YEAR	M	K	J	R	ℓ	E	u	$\Delta \ell$	Δu
	1								
	2								
	3								
	3*								
	3*								
	3*								
	3*								
	3*								
	3*								

WORKSHEET NO. 4.2 **Confidence Level** ________ **V(B) =** ________ , **V =** ________

FIFTH SCHOOL YEAR ________

1	2	3	4	5	6	7	8	9	10
YEAR	M	K	J	R	ℓ	E	u	$\Delta\ell$	Δu
	1								
	2								
	3								
	4								
	4*								
	4*								
	4*								
	4*								
	4*								
	4*								

SIXTH SCHOOL YEAR ________

1	2	3	4	5	6	7	8	9	10
YEAR	M	K	J	R	ℓ	E	u	$\Delta\ell$	Δu
	1								
	2								
	3								
	4								
	5								
	5*								
	5*								
	5*								
	5*								
	5*								

SEVENTH SCHOOL YEAR ________

1	2	3	4	5	6	7	8	9	10
YEAR	M	K	J	R	ℓ	E	u	$\Delta\ell$	Δu
	1								
	2								
	3								
	4								
	5								
	6								
	6*								
	6*								
	6*								
	6*								

EIGHTH SCHOOL YEAR ________

1	2	3	4	5	6	7	8	9	10
YEAR	M	K	J	R	ℓ	E	u	$\Delta\ell$	Δu
	1								
	2								
	3								
	4								
	5								
	6								
	7								
	7*								
	7*								
	7*								

WORKSHEET NO. 4.3 **Confidence Level** ________ **V(B) =** ________ , **V =** ________

NINTH SCHOOL YEAR ________

1	2	3	4	5	6	7	8	9	10
YEAR	M	K	J	R	ℓ	E	u	$\Delta\ell$	Δu
	1								
	2								
	3								
	4								
	5								
	6								
	7								
	8								
	8*								
	8*								

TENTH SCHOOL YEAR ________

1	2	3	4	5	6	7	8	9	10
YEAR	M	K	J	R	ℓ	E	u	$\Delta\ell$	Δu
	1								
	2								
	3								
	4								
	5								
	6								
	7								
	8								
	9								
	9*								

ELEVENTH SCHOOL YEAR ________

1	2	3	4	5	6	7	8	9	10
YEAR	M	K	J	R	ℓ	E	u	$\Delta\ell$	Δu
	1								
	2								
	3								
	4								
	5								
	6								
	7								
	8								
	9								
	10								

TWELFTH SCHOOL YEAR ________

1	2	3	4	5	6	7	8	9	10
YEAR	M	K	J	R	ℓ	E	u	$\Delta\ell$	Δu
	1								
	2								
	3								
	4								
	5								
	6								
	7								
	8								
	9								
	10								

55

IF INITIAL SCHOOL YEAR ON WORKSHEET NO. 1 IS KINDERGARTEN OR AGE 5, ONE MORE SECTION (WORKSHEET NO. 4.4) WILL BE NEEDED.

WORKSHEET NO. 4.4 **Confidence Level** _______ **V(B) =** _______ , **V =** _________

THIRTEENTH SCHOOL YEAR ________									
1	2	3	4	5	6	7	8	9	10
YEAR	M	K	J	R	ℓ	E	u	$\Delta\ell$	Δu
	1								
	2								
	3								
	4								
	5								
	6								
	7								
	8								
	9								
	10								

WORKSHEET NO. 5 **CONFIDENCE LIMITS FOR GROUPS – TOTALS**

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
YEAR	STATIS-TIC	INITIAL SCHOOL YEAR	GRADE or AGE												Σ	$\sqrt{\Sigma\Delta^2}$	CONF. LIMITS
a	E																
___ b	$\overline{\Delta u}^2$																u
c	$\overline{\Delta \ell}^2$																ℓ
a	E																
___ b	$\overline{\Delta u}^2$																u
c	$\overline{\Delta \ell}^2$																ℓ
a	E																
___ b	$\overline{\Delta u}^2$																u
c	$\overline{\Delta \ell}^2$																ℓ
a	E																
___ b	$\overline{\Delta u}^2$																u
c	$\overline{\Delta \ell}^2$																ℓ
a	E																
___ b	$\overline{\Delta u}^2$																u
c	$\overline{\Delta \ell}^2$																ℓ
a	E																
___ b	$\overline{\Delta u}^2$																u
c	$\overline{\Delta \ell}^2$																ℓ
a	E																
___ b	$\overline{\Delta u}^2$																u
c	$\overline{\Delta \ell}^2$																ℓ
a	E																
___ b	$\overline{\Delta u}^2$																u
c	$\overline{\Delta \ell}^2$																ℓ
a	E																
___ b	$\overline{\Delta u}^2$																u
c	$\overline{\Delta \ell}^2$																ℓ
a	E																
___ b	$\overline{\Delta u}^2$																u
c	$\overline{\Delta \ell}^2$																ℓ

APPENDIX

UNDERLYING ASSUMPTIONS OF CONFIDENCE LIMITS

Information required to determine the confidence limits for an individual projection by the cohort survival method includes survival rates—both Birth Survival Rates and Grade (Age) Survival Rates—the number of years of historical enrollment data employed in making the projection (N) and a decision about the degree of confidence desired.

N and the survival rates are used to obtain an estimate of survival rate variance or, more precisely, the variance of the natural logarithms of these rates.

Relative survival rates are calculated first (the ratio of a survival rate to the average survival rate of the column of which it is a member). In order to avoid dependence among the relative survival rates, one is dropped from consideration in each column. Since the data from the initial year is probably least relevant for predictions, it is the one generally chosen to be eliminated.

The next step involves determining the specific Relative Birth Survival Rates and the Relative Grade (Age) Survival Rates which fall most closely to the 7th percentile and 93rd percentile points of each of the groups of rates if they were arranged in ascending order. The rate nearest the 7th percentile is designated C_1. The rate nearest the 93rd percentile is designated C_2.* A ratio of these specific rates, C_2/C_1, is designated W and is used in the formula below to calculate the variance. [W(B) as used in the text is a shorthand for designating the ratio, W, associated with the Relative *Birth* Survival rates.]

The final step in determining the variance (V) is to apply the computed value of W [or W(B)] and N to Table 1 and read it off. The formula used in Table 1 is:

$$V = \frac{N}{N-1}(.78 \log_{10} W)^2 \quad \text{where} \quad W = \frac{C_2}{C_1}$$

Because of the time factor between births and the initial grade of school, a corrective multiplier, 1.33, is applied to the value of V computed by the formula:

*If the number of relative survival rates involved in the procedures is less than 15, a normal situation for the Relative Birth Survival Rates, the 7th and 93rd percentile rates are the smallest and largest respectively.

$$V(B) = 1.33\ V \text{ where } V \text{ is determined from } W(B) = \frac{C_2(B)}{C_1(B)}$$

Knowing the variance associated with Birth Survival Rates, V(B), and that associated with Grade Survival Rates, V, one may then proceed to apply these data to the determination of confidence limits for individual projections. Those limits are dependent on the variances, on the choice of confidence desired and on the number of prior projections compounded to derive the specific one for which the individual limits are desired. On this latter point, for example, a projection for Grade 3 next year will be based on this year's data. Only one projection is required. A projection for Grade 3 two years hence is based on this year's data for Grade 1, and next year's projected Grade 2 enrollments—two projections compounding the errors inherent in each. Five years from now a projection for Grade 3 will include projections for Grades 1, 2, and 3, and a projection for Kindergarten from birth data—four projections, one from the other. The number of such one-from-the-other projections intervening between a known quantity and the specific projection in question is the value of M on Worksheet 4.

To determine M where kindergarten projections are used, compare the grade level and the number of years ahead for which the prediction is made. The smaller number is M:

$$M = \left\{ \begin{matrix} \text{grade level} \\ \text{years projected} \end{matrix} \right\} \text{ whichever is smaller}$$

Where no kindergartens are involved:

$$M = \left\{ \begin{matrix} \text{grade level - 1} \\ \text{years projected} \end{matrix} \right\} \text{ whichever is smaller}$$

M and N determine the value of K, using Table 2, for a chosen level of confidence (50%, 80%, 90%, or 95%). In turn the value of K combined with variances V and V(B) produce a confidence ratio R, which may be applied to the specific individual projection to determine the upper and lower confidence limits.

In Table 2, the values of K have been calculated for each confidence level from the following formulas:

$$\text{For 95\% confidence } K_{M,N} = (1.96)^2 \frac{M+N}{N}$$

$$\text{For 90\% confidence } K_{M,N} = (1.645)^2 \frac{M+N}{N}$$

$$\text{For 80\% confidence } K_{M,N} = (1.282)^2 \frac{M+N}{N}$$

$$\text{For 50\% confidence } K_{M,N} = (0.674)^2 \frac{M+N}{N}$$

The confidence ratio, R, is determined from the product of $K_{M,N}$ and the variance V. For projections stemming from births as the "known quantity," the confidence ratio is also dependent on the birth variance, V(B), and the factor $K_{1,N}$. Letting J represent these products:

$$J_{M,N} = V \times K_{M,N} \text{ where grade level* > or = years projected, or}$$

$$J_{M,N} = V \times K_{M,N} + V(B) \times K_{1,N} \text{ where grade level* < years projected.}$$

Then

$$R = e\sqrt{J_{M,N}}$$

The values in Table 3 have been calculated from this formula.

Given an estimated enrollment, E, then one may assert with confidence of 50%, 80%, 90%, or 95% that the true enrollment will lie between 1/R x E, the lower limit, and R x E, the upper limit.

Confidence limits for a group of enrollments (subtotals and totals) are determined by standard procedures:

$D = \sqrt{\Sigma d^2}$ where

D = deviation for the group (total)

d = deviation of the confidence limits for individual estimates involved.

*If birth survival rates are computed from birth to Grade 1, use *grade level - 1* in place of grade level.